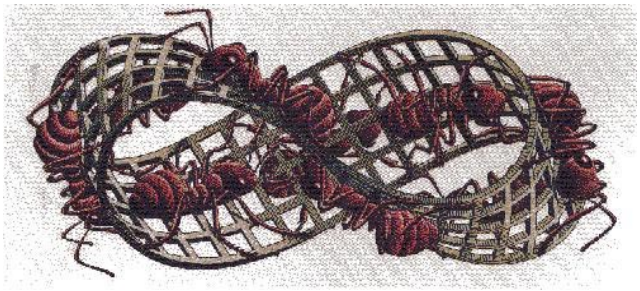
Activity #11: Investigation of the Moebius Strip (Teacher version)
Math

Note: Students must work in pairs for this activity and record one set of responses on the worksheet. As you work in pairs, you may want to discuss your results with other teams.



by M. C. Escher

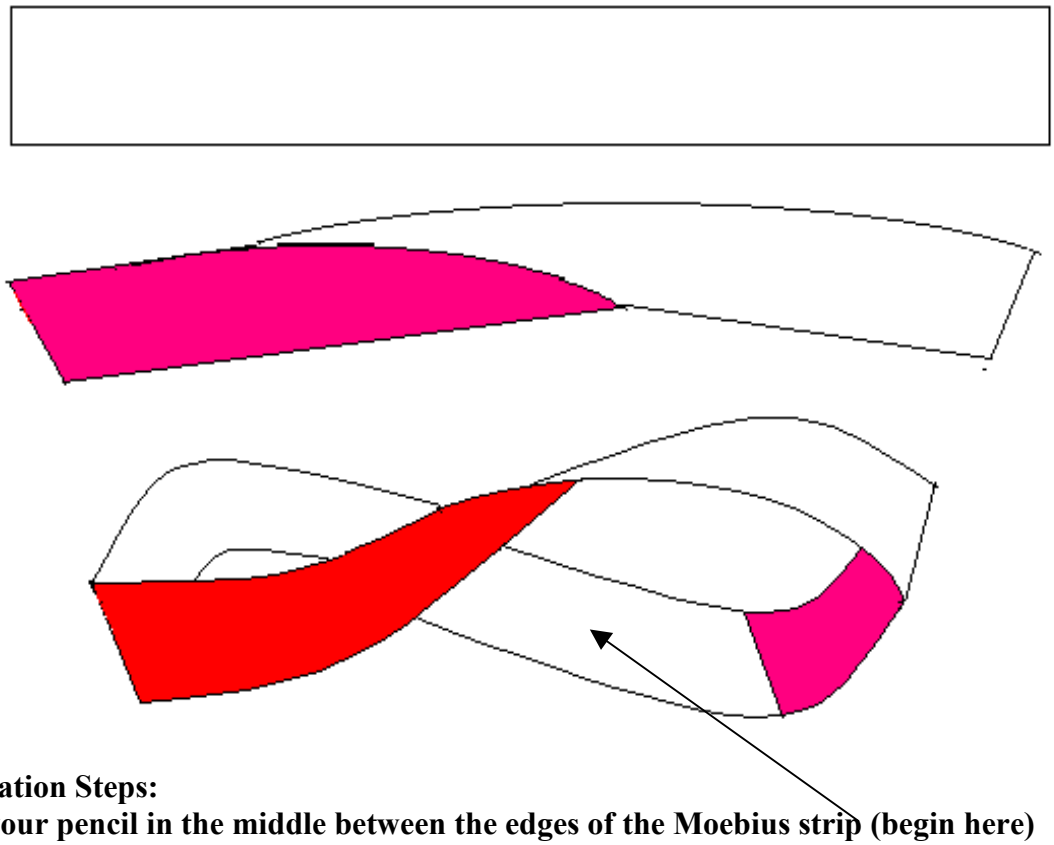
National Standards addressed:
Content Standards:
Geometry Expectation: Students will analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
Process Standards:
Problem Solving Expectation: Students will build new mathematical knowledge through problem solving.
Reasoning and Proof Expectation: Students will make and investigate mathematical conjectures.
Communication Expectations: Students will organize and consolidate their mathematical thinking through communication; students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Purpose: **To begin investigation of topology**
To guess conclusions given hypotheses
To experiment to prove conclusions valid (or invalid)
To use previously learned knowledge

Materials: graph paper (4 units per inch), tape or stapler, red crayons

Procedure:

1. Cut 12 strips, each 11" long and one inch wide from the graph paper.
2. Take one strip and color one side red.
3. Take this strip, make a loop with it, and turn one end over before taping the ends together. You have constructed a Moebius strip or band.



Investigation Steps:

1. Put your pencil in the middle between the edges of the Moebius strip (begin here) and draw a line down the center. Stop when you get back to that same point. You should notice that you do not go over the edge of the strip on your move back to the starting point. You have only 1 side!
2. Take this Moebius strip and cut parallel to the edge about in the middle, right along the line you just drew. What happens? _____

_____

_____

_____

(Students find one loop with four half-twists. The new loop is twice as long and half as wide.)

3. Make another Moebius strip and cut parallel to the edges about one-third of the way from one edge. What do you think will happen? _____

_____

_____

_____

Describe what happens. _____

_____

_____

_____

(Students find two intertwined loops. One is a Moebius strip, the same length as the original, but only one-third as wide. The second loop has four half-twists and is twice as long and only one-third as wide as the original Moebius strip.)

4. Make another Moebius strip and cut parallel to the edges about one-fourth of the way from one edge. What do you think will happen? _____

_____

_____

_____

Describe what happens. _____

_____

_____

_____

(Students find two intertwined loops. One is a Moebius strip, the same length as the original, but only one-half as wide. The second loop has four half-twists and is twice as long and only one-fourth as wide as the original Moebius strip.)

5. What would happen if you cut another Moebius strip parallel to the edge only this time, one-fifth of the way from one edge? Are you sure? If not, verify.

_____

_____

_____

(Students find two intertwined loops. One is a Moebius strip, the same length as the original, but only three-fifths as wide. The second loop has four half-twists and is twice as long and only one-fifth as wide as the original Moebius strip.)

6. Make a loop with two half-twists. Put your pencil in the middle between the edges of loop and draw a line down the center. Stop when you get back to that same point. How many sides does this loop have? _____**(One)**

7. Take this loop and cut parallel to the edge about in the middle, right along the line you just drew. What happens? _____

_____

_____

(Students find two intertwined loops half as wide and same length as original, both with two half-twists.)

8. Make another loop with two half-twists. Take this loop and cut parallel to the edges about one-third of the way from one edge. What happens? _____

_____

_____

_____

(Students find two intertwined loops, both with two half-twists, but one loop is twice as wide as the other is.)

9. Make a loop with three half-twists. Put your pencil in the middle between the edges of loop and draw a line down the center. Stop when you get back to that same point. How many sides does this loop have? _____**(One)**

10. Take this loop and cut parallel to the edge about in the middle, right along the line you just drew. What happens? _____

_____

_____

(Students find one loop with a knot!)

11. Make another loop with three half-twists. Take this loop and cut parallel to the edges about one-third of the way from one edge. What happens? _____

_____

_____

(Students find two intertwined loops each one-third as wide as the original. One loop is the same length and has three half-twists. The other loop is twice as long as the original and like the results found in #9.)

12. Make another loop with three half-twists. First, try to guess what happens when you cut this loop one-fourth of the way from an edge. Then, take this loop and cut it. What happens?

_____

_____

_____

(Students find two intertwined loops as in #10. One loop is half as wide and as long as the original and has three half-twists. The other loop is twice as long and one-fourth as wide and has a knot like the results found in #9.)

Follow-up Activities:

1. Write a one-page paper describing what you experienced while you worked on the activity.
2. Look up "topology" and prepare a short speech to deliver to class, using visuals or manipulatives if necessary to help clarify meaning.
3. Prepare a report on M. C. Escher.
4. Prepare a report on Augustus Ferdinand Moebius.
5. Find evidence of the practical uses of Moebius strips and present your findings to classmates in a short speech.

The following web sites and articles provide enrichment and support for this activity:

1. http://www-groups.dcs.st-and.ac.uk:80/~history/Mathematicians/Mobius.html
2. http://mathforum.org/sum95/math_and/moebius/moebius.html
3. http://www.cut-the-knot.com/do_you_know/moebius.shtml
4. http://humber.northnet.org/weeks/
5. http://www.math.wayne.edu/~rrb/topology.html
6. http://www.math.twsu.edu/history/activities/geometry-act.html#mob-act
7. http://dimacs.rutgers.edu/drei/96/classroom/topology/lessons/magicring.html
8. **GEOMETRY AN INTEGRATED APPROACH, by Larson, Boswell, Stiff, D. C. Heath and Company ©1995.**